

In arriving at the results of this study, entry and vehicle conditions were specified. Generality is not lost, however, since Slye<sup>3</sup> shows that a turn is relatively insensitive to variations in re-entry lift loading ( $W/SC_L$ ) and initial flight conditions if it is initiated at speeds equal to or slightly in excess of the original orbital velocity. Therefore, the results of this study may be considered applicable for any reasonable  $W/SC_L$  and for return from any low-altitude orbit.

Entry trajectories were computed by integrating the complete equations of motion on an IBM 7090 computer. Characteristics selected were as follows. De-orbit was accomplished with an impulse of 95 fps from an equatorial, circular orbit at an 80-naut-mile altitude. A nonrotating spherical earth model was used and a value of 100 psf specified for  $W/SC_L$ .

An  $L/D$  of 3.6 was found to be adequate for global coverage when advantage is taken of a varying bank-angle schedule. With a constant 45° bank angle (schedule  $D$ ), an  $L/D$  of about 6.5 would be required.

### References

- 1 Bryson, A. E. and Denham, W. F., "A steepest-ascent method for solving optimum programming problems," *J. Appl. Mech.* **29**, 247-257 (June 1962).
- 2 Bryson, A. E., "Optimum lateral turns for a re-entry glider," *Aerospace Eng.* **21**, 18-23 (March 1962).
- 3 Slye, R. E., "An analytical method for studying the lateral motion of atmospheric entry vehicles," NASA TND-325 (September 1960).

## Method of Determining Saturated Liquid and Saturated Vapor Entropy

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### Nomenclature

- $C_{SL}$  = specific heat under saturated liquid conditions  
 $C_p$  = specific heat at constant pressure  
 $H$  = enthalpy  
 $\Delta H_{LV}$  = latent heat of vaporization  
 $P$  = pressure  
 $S$  = entropy  
 $\Delta S_{LV}$  = latent entropy of vaporization  
 $T$  = absolute temperature  
 $T_c$  = critical temperature  
 $V$  = specific volume

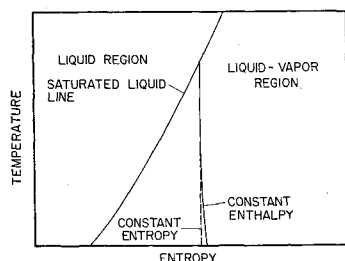


Fig. 1 Saturated liquid region.

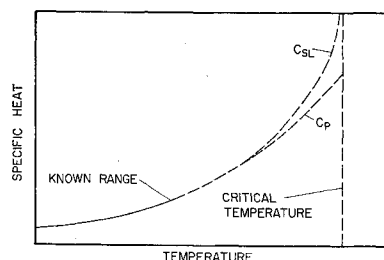


Fig. 2 Extrapolation of specific heat.

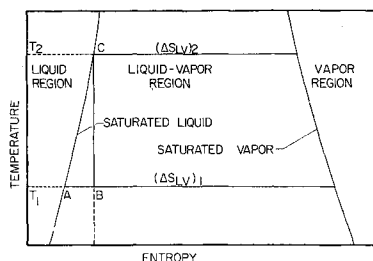


Fig. 3 Construction of temperature-entropy diagram.

### Subscripts

- $H$  = constant enthalpy process  
 $S$  = constant entropy process  
 $SL$  = saturated liquid conditions

### I. Introduction

THE present lack of thermodynamic data available for all but the most common liquid propellants makes difficult the analysis of physical processes, such as the expansion of saturated liquids in flow-control nozzles. The following method is presented as a means of determining entropies in the liquid-vapor region, for temperatures not exceeding the critical point. The resulting temperature-entropy diagrams have been found to be in excellent agreement with accepted diagrams and with those computed by far more rigorous techniques.

### II. Analysis

It can be shown from basic thermodynamics that<sup>1</sup>

$$dH = TdS + VdP \quad (1)$$

The relative magnitude of the  $VdP$  term for liquids is so small (less than 0.5% of  $dH$  for saturated liquid water at 150 psia and 358°F) that it may be neglected with small error, and the following is obtained:

$$dH = TdS \quad (2)$$

From this it follows that, if either entropy or enthalpy is constant, its derivative will reduce to zero, and the other must also be constant, and so it may be written as

$$(\partial H)_S = 0 \quad (3)$$

$$(\partial S)_H = 0 \quad (3a)$$

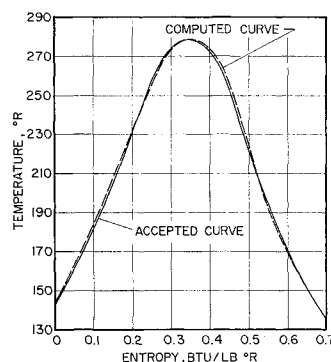


Fig. 4 Comparison of calculated and accepted temperature-entropy diagrams for oxygen.

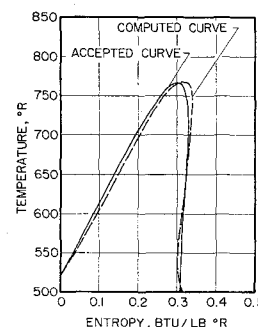


Fig. 5 Comparison of calculated and accepted temperature-entropy diagrams for  $n$ -butane.

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Fig. 6 Thermodynamic properties of Aerozine 50.

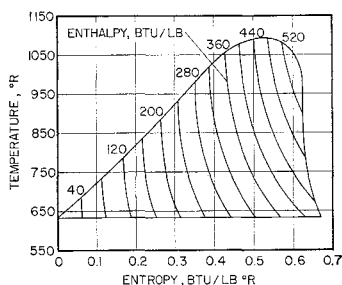


Fig. 7 Thermodynamic properties of chlorine trifluoride.

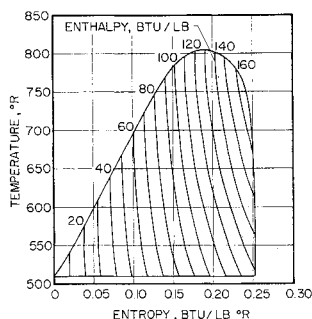


Fig. 8 Thermodynamic properties of monomethylhydrazine.

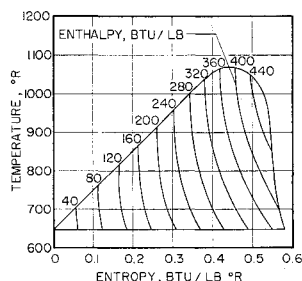
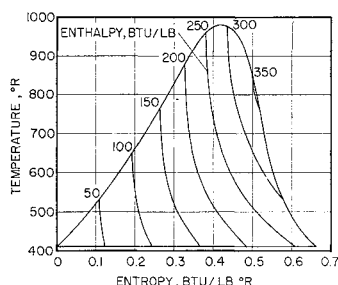


Fig. 9 Thermodynamic properties of red fuming nitric acid (0.15 NO<sub>2</sub>).



If a temperature-entropy diagram is constructed where a vertical line is a path of constant entropy, this vertical line must therefore be also a path of constant enthalpy in any region of the diagram where Eq. (3) holds. Such a region is that along the saturated liquid line shown in Fig. 1.

For liquids,  $H$  and  $S$  depend on  $T$  and are essentially independent of  $P$ , as long as the critical point is not approached. Therefore,  $C_p = (\partial H)/(\partial T)_{SL}$  for temperatures well below  $T_c$ . The enthalpy of the saturated liquid may then be computed for specific heat data. The specific heat of a saturated liquid approaches infinity at the critical point; this fact is of value in extrapolating measured specific heat, as shown in Fig. 2.

The available enthalpy data, in the form of specific heat and heat of vaporization for a range of temperatures up to the critical temperature, may be used to construct a temperature-entropy diagram, as illustrated in Fig. 3.

From saturated liquid specific heat data,

$$\Delta H_{AC} = (C_p)_{1-2}(T_2 - T_1) = H_C - H_A = (H_{SL})_2 - (H_{SL})_1 \quad (4)$$

As long as point  $B$  is close enough to the saturated liquid

Fig. 10 Thermodynamic properties of nitrogen tetroxide.

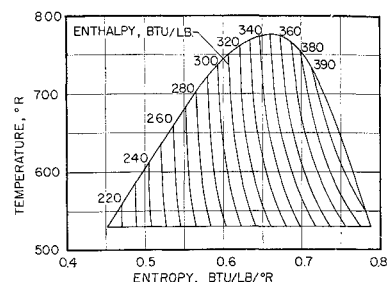


Fig. 11 Thermodynamic properties of nitrogen trifluoride.

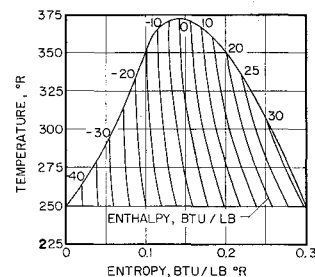


Fig. 12 Thermodynamic properties of oxygen difluoride.

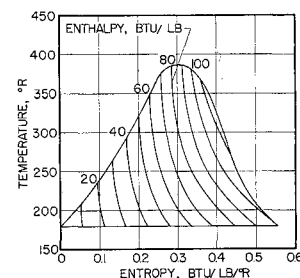
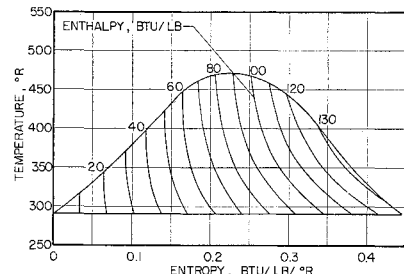


Fig. 13 Thermodynamic properties of ozone.



line, line  $BC$  is vertical,  $(\partial H)_S = 0$ , and

$$\Delta H_{BC} = 0 \quad (5)$$

The actual calculations were performed on an IBM 7090 computer with a  $\Delta T$  of  $0.5^\circ\text{F}$ . It is apparent from Fig. 3 that

$$\Delta H_{AC} = \Delta H_{AB} \quad (6)$$

The partial entropy of vaporization between  $A$  and  $B$  at  $T_1$  is

$$\Delta S_{AB} = \Delta H_{AB}/T_1 \quad (7)$$

It is also apparent from Fig. 3 that

$$\Delta S_{AC} = \Delta S_{AB} \quad (8)$$

Substituting and combining, it is shown that

$$\Delta S_{AC} = [(H_{SL})_2 - (H_{SL})_1]/T_1 \quad (9)$$

Relative entropies along the saturated liquid line can then be easily computed if the enthalpies are known.

For complete vaporization,

$$\Delta S_{LV} = \Delta H_{LV}/T \quad (10)$$

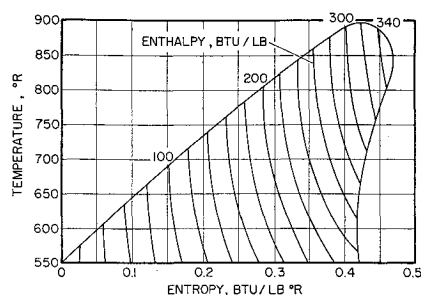


Fig. 14 Thermodynamic properties of pentaborane.

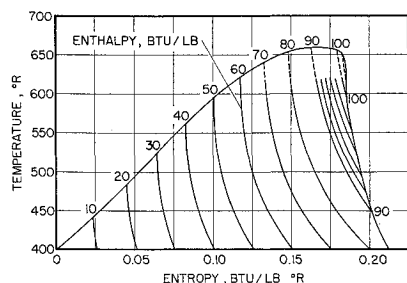


Fig. 15 Thermodynamic properties of perchloryl fluoride.

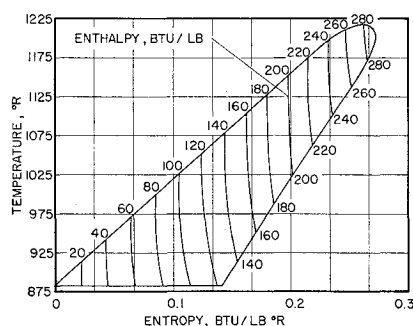


Fig. 16 Thermodynamic properties of RP-1 ( $H/C = 2.0$ ).

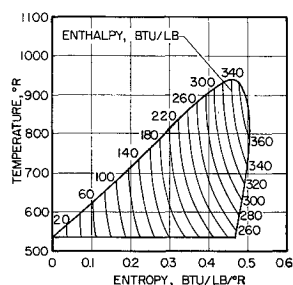


Fig. 17 Thermodynamic properties of unsymmetrical dimethylhydrazine.

This makes it possible to compute vapor entropies, and thus the temperature-entropy diagram can be constructed if liquid specific heat and heat of vaporization data are available.

The temperature-entropy diagrams computed by this method for oxygen and *n*-butane are compared with those given in the literature (Figs. 4 and 5).

The temperature-entropy diagrams of 12 liquid propellants were computed by this method and are given in Figs. 6-17.<sup>2,3</sup>

#### References

- <sup>1</sup> Dodge, B. F., *Chemical Engineering Thermodynamics* (McGraw-Hill Book Co. Inc., New York, 1944), pp. 103-201.
- <sup>2</sup> "Nitrogen tetroxide," Allied Chemical Product Bull., Nitrogen Div., New York, p. 30.
- <sup>3</sup> Petrozzi, P. J. and Dean, L. E., "Physical properties of liquid propellants," Aerojet-General Corp., Sacramento, Calif., Rept. LRP-178 (July 1960).

## Design of a High Enthalpy, Radio Frequency, Gas Discharge Volume

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A design of a highly water-cooled copper-walled discharge volume that allows radio frequency generation of high enthalpy, dense plasmas is described. It is noted that this design allows steady generation of plasmas of substantially higher enthalpy level than have previously been able to be obtained with a quartz-walled discharge volume, either cooled or uncooled.

CURRENTLY, most steady, thermal plasmas are generated by electric arc discharges. The major problems associated with this method of gas heating are well known: 1) plasma contamination with eroded electrode material; 2) limited selection of gases that may be heated due to chemical erosion of electrode surfaces; and 3) spatially non-uniform heating process.

Recently, work has begun at a few laboratories<sup>1,2</sup> in the United States on induction heating of plasmas. Briefly, induction heating of plasmas is based on the principle that eddy currents may be induced in an electrical conductor from an external, oscillatory circuit. These induced currents meet resistance to their flow, and joule heating of the conductor occurs. Normally a gas is a very good electrical insulator. However, upon application of a spark or introduction of a source of secondary electrons, the insulating properties of the gas begin to break down, and it becomes a fairly good electrical conductor. The electromagnetic energy being carried in an external oscillatory circuit may then couple to the small volume of conducting gas and enlarge it. Provided that the discharge is properly stabilized with respect to power level and aerodynamic cooling, a steady state discharge of high-power level will occur. This method of gas heating shows substantial promise of alleviating many of the problems associated with electric arc heating of gases.

Most of the current work in the radio frequency heating of gases has used a quartz walled discharge volume, either with or without water cooling, to contain the heated gas. Quartz is a very good dielectric with a very low loss factor; however, its melting point and mechanical strength severely limit the enthalpy level that may be obtained in the heated gas. It has been found<sup>1-3</sup> that an argon plasma having an enthalpy of about 50 kcal/g-mole is about the maximum that may be generated in a quartz discharge volume before melting occurs. Adequate water cooling of the quartz is limited by its thermal conductivity and mechanical strength. Thus, it appears that a quartz wall is not feasible for containing the higher enthalpy plasmas required in hyperthermal wind tunnels, MHD power generators, electrical propulsion, and first stages of controlled fusion processes.

A novel solution to this problem is the highly water-cooled copper wall that has found extensive application in electric arc gas heaters. A highly water-cooled copper wall with its high thermal conductivity and good mechanical strength has been found to be able to contain plasmas having enthalpy levels of hundreds of kcal/g-mole. However, for rf heating of gases, the copper wall must contain a slit so as not to shield the gas, i.e., the slit prevents closed current loops from occurring in the water jacket wall which tend to dissipate all the incident electromagnetic power.

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